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ABSTRACT

The question of least-squares weights versus equal weights has been a subject of great interest to researchers for over 60 years. Several researchers have compared the efficiency of equal weights and that of least-squares weights under different conditions. Recently, S. V. Paunonen and R. C. Gardner stressed that the necessary and sufficient condition for equal-weights aggregation is that the predictors satisfy the requirements of psychometric parallelism. In this study, the effect of psychometric parallelism on the error of accuracy for equal weights and least-squares weights was investigated with the combination of different numbers of predictors, sample sizes, and intercorrelations. The findings indicate that equal weights always perform more precisely than least-squares weights as long as the following situations are satisfied: (1) the number of predictors is small; (2) the ratio of observation to predictor is small, less than or equal to 10; and (3) the magnitude of the mean intercorrelation is high, at least 0.6. Least-squares weights may perform more accurately than equal weights in the opposite situations of a large number of predictors, a high ratio of observation to predictor, and low intercorrelations. Nevertheless, the combination of a large number of predictors, large sample sizes, and a low mean of intercorrelation does not guarantee that least-squares weights are more accurate than equal weights. Equal weights are still more accurate than least-squares weights for the sample with a relatively high level of psychometric parallelism. (Contains 16 tables and 34 references.) (Author/SLD)

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The Error of Accuracy for Two Regression Techniques: Does Psychometric Parallelism Matter?

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Abstract

The question of least-squares weights vs. equal weights has been a subject of great interest to researchers for over sixty years. Several researchers have compared the efficiency of equal weights and that of least-squares weights under different conditions. Recently, Paunonen and Gardner stressed that the necessary and sufficient condition for equal-weights aggregation is that the predictors satisfy the requirements of psychometric parallelism. In this study, the effect of psychometric parallelism on the error of accuracy for equal weights and least-squares weights was investigated with the combination of different number of predictors, sample size, and intercorrelations. The findings indicate that equal weights always perform more precisely than least-squares weights as long as the following situations are satisfied: (a) the number of predictors is small, 3 , (b) the ratio of observation to predictor is small, less than or equal to 10, and (c) the magnitude of the mean of intercorrelation is high, at least 0.6. Least-squares weights may perform more accurately than equal weights with the opposite combination: (a) a large number of predictors, (b) high ratio of observation to predictor, and (c) low intercorrelations. Nevertheless, the combination of a large number of predictors, large sample sizes, and a low mean of intercorrelation does not guarantee that least-squares weights are more accurate than equal weights. Eequal weights are still more accurate than least-squares weights for the sample with relatively high level of psychometric parallelism.

The Error of Accuracy for Two Regression Techniques: Does Psychometric Parallelism Matter?

An important aspect of any multiple regression analysis is that of determining how the predictors entering into the composite are to be weighted. (Lawshe & Schucker, 1959; Wang & Stanley, 1970). Least-squares weights, also called ordinary least-squares weights, are most commonly used in weighting the predictors into a composite (McCormick & Ilgen, 1980). When a linear regression equation with least-squares weights is used to create a score, $\hat{Y}_i$, to predict a score, Y_i, for individual, i, in the sample size, n, the accuracy of the prediction is best in one sense: the sum over individuals of the squared deviations of Y from $\hat{Y}_i$ is minimized. That is, a least-squares equation minimizes the sum of squared errors, $\Sigma_{i=1}^{n}(Y_i - \hat{Y}_i)^2$, and maximizes the linear correlation between criterion variable and composite scores (Allen & Yen, 1979; Kromrey & Hines, 1993; Vogt, 1993).

Some difficulties with using least-squares weights in prediction are: (a) they consume degrees of freedom in the estimation of those weights; (b) they lead to a decrease in effectiveness when sampling procedures are poor (Einhorn & Hogarth, 1975); (c) they often cause a shrinkage in practical situations when the initial sample is small (Dorans & Drasgow, 1978; Schmidt, 1971); and (d) they can not perform when the criterion variable is not available (Wilks, 1938). An alternative predictor-weighting shceme to least-squares weights is equal weights (Tatsuoka, 1988). The procedure of equal weights, according to Srinivasan (1977), is as follows:

> (a) orient each of the *k* predictors such that the greater the value of a predictor (with values for the other predictors remaining the same), the greater will be the criterion;

(b) scale each of the *k* predictors into standardized form (i.e., zero mean and unit variance);
(c) form a single "composite predictor" by simply adding up the *k* predictor scores;
(d) run a simple regression with the composite predictors as the only independent variable (aside from an intercept or constant term);
(e) use the regression equation estimated in (d) to predict the criterion values for future samples. (p. 1)

Research has indicated that the predictive power of equal weights is as well as the least-squares approach (Davis & Sauser, 1991). Wesman and Bennett (1959) investigated the efficiency of equal weights and that of least-squares weights in predicting the first term GPA of college freshmen by combining scores from the verbal, numerical, and information tests of the College Qualification Tests. Students from four schools were included in the study with sample sizes ranging from 76 to 449. Each equation of least-squares weights was then cross-validated in each of the other schools. The resulting validity coefficients did not appear to differ from those obtained by the use of equal weights.

A study by Lawshe and Schucker (1959) compared the predictive efficiencies among four different weighting methods: standard deviation of each predictor, inverse of standard deviation, least-squares weights, and equal weights. The performance of these weighting methods was compared by combining aptitude test and high school achievement data to predict success of first year engineering students. The criterion for success was the minimum GPA required for students to continue in the school of engineering. Three analyses of variance were performed to test for differences between the predictive efficiency of the four methods at different sample sizes. It was concluded that least-squares weights were no more efficient than equal weights with samples of less than 100.

Trattner (1963) examined three methods of selecting and weighting test for a test battery. The test selection and weighting methods included the Wherry-Gaylord Integral Gross Score method, the Civil Service Commission Job Analysis Method, and the General Blue Collar Test Battery. The first of these involved least-squares weights based on the correlations of tests with a criterion and intercorrelations of the test with other tests. The other two methods applied equal weights to the tests. All three methods were applied to test and performance appraisal data for journeyman employees in 12 different job categories. Sample sizes in these categories ranged from 130 to 250. The regression equations obtained from each job category were then cross-validated on a sample from a different job category. Again, equal weights appeared to be as effective as least-squares weights.

In 1978, Schmidt, Johnson, and Gugel evaluated the application of linear policy-capturing models to the real-world decision task of graduate admission in the psychology department at Michigan State University during 1967-1971. Indices used as predictors of admission decisions were the three GRE scores (Verbal, Quantitative, and Advanced) and undergraduate GPA for the junior and senior years. The results of the study indicated that use of equal weights of GRE scores and GPA in admission decisions could be expected to be at least as effective as least-squares weights.

In examining the relationship and superiority among the weighting methods for combining several predictors into a composite, Aamodt and Kimbrough (1985) evaluated four methods in their studies: critical incident, rank order, equal weights, and least-squares weights. The results of their studies indicated that there is no significant difference among these four methods in terms of resistance to validity shrinkage. However, even though the

differences were not significant, more shrinkage did occur when least-squares weights were used than when any other method was used.

Silverstein (1987) employed data from three standardization samples for the WISC-R (Wechsler Intelligence Scale for Children-Revised), the WPPSI (Wechsler Preschool and Primary Scale of Intelligence), and the WAIS-R (Wechsler Adult Intelligence Scale-Revised) to compare the validities and reliabilities of each of several short forms, using least-squares weights and equal weights. The validities and reliabilities varied little from one set of weights to another, so that a strong case could be made for the use of equal weights, which also possess the advantages of simplicity and robustness.

Recently, Muchinsky, and Skilling (1992) assessed the economic utility of five weighting methods (chi-square, weighted application blank, Bayes, equal weights, and least-squares weights) for evaluating consumer loan applications. A sample of 443 consumer loans which had been classified as either good or bad accounts was analyzed by 11 predictor variables. Cross-validation correlations revealed that consumer credit risk is highly predictable. The degrees of shrinkage between validation and cross-validation samples were 0.02 and 0.08 for equal weights and least-squares weights, respectively. The proportion of hits (correct classifications) was 80.2 for equal weights, and only 72.1 for least-squares weights. Biserial correlation coefficients between the predicted status (accept versus reject) and actual credit risk status (good versus bad) were also computed. The correlation coefficients were 0.54 for equal weights and 0.44 for least-squares weights.

The results of these studies consistently indicated that equal weights can often be as efficient in prediction as least-squares weights. Besides the empirical investigations, several researchers have compared the efficiency of equal weights and that of least-squares

weights under different conditions. Wilks (1938) studied the effect of the number of predictors and the intercorrelation among predictors on the relationship between these two weighting models mathematically. Einhorn and Hogarth (1975) used the ratio of observation to predictor, the definition of the criterion variable, and the mean of the intercorrelation as factors in examining equal weights for decision making. Wainer (1976) employed the range of the beta weights to demonstrate the robustness of equal weights. Zimmerman (1983) studied the effectiveness and the interaction of the mean of validities and the variability among validities in comparing the accuracy of equal weights, least-squares weights, and ridge regression. According to Perloff and Persons (1988), the magnitude of the intercorrelation among predictors is the key in determining when to use equal weights instead of least-squares weights. In 1991, Paunonen and Gardner stressed that the necessary and sufficient condition for equal-weights aggregation is that the predictors satisfy the requirements of psychometric parallelism.

What is psychometric parallelism? According to Lord and Novick (1968), the definition of psychometric parallelism includes two conditions. First, submeasures or items must be random samples from a population or domain of measures producing the same true scores on the attribute being measured. Second, error score variables must be equal but uncorrelated. If these two conditions exist, the observed scores will have equal expected values for their means, standard deviations, intercorrelations, and correlations with external variables.

A test of psychometric parallelism among predictors proposed by Wilks (1946) is given by:

$$L = \frac{D}{\bar{s}^2[1+(k-1)\bar{r}][\bar{s}^2(1-\bar{r})+v]^{k-1}} \quad (1)$$

where L is the likelihood ratio for testing the hypothesis that the means are equal, the variances are equal, and the covariances are equal, k is the number of predictors, D is the determinant of the sample variance-covariance matrix of predictors, $\bar{s}^2$ is the average variance of predictors, $\bar{r}$ is the average of intercorrelations among predictors, and v is the variance of predictors' means. For n examinees, the quantity of $-n \log_e L$ is distributed approximately as χ^2 with $(k/2)(k + 3) - 3$ degrees of freedom when the null hypothesis of equal means, variances, and covariances for predictors is true. That is, the predictors are statistically parallel.

If all the variables are expressed in standard-score form, then the first equation can be rewritten as follows because $\bar{s}^2$ is equal to 1 and v is equal to 0.

$$L = \frac{D}{[1+(k-1)\bar{r}](1-\bar{r})^{k-1}} \quad (2)$$

Obviously, the above equation is a test of the equality of intercorrelations among predictors (Paunonen & Gardner, 1991). The numerator of this equation is a standardized variance-covariance matrix. A distinguishing property of a variance-covariance matrix is positive definiteness (Worthke, 1993). The determinant of a positive definite matrix is always positive (Graybill, 1961). The range of the value of L is always positive but less than or equal to 1 (Wilks, 1946). When all intercorrelations among predictors are not statistically different from one another, the degree of psychometric parallelism among them is strong. Namely, when L is equal or close to 1, the χ^2 value for testing psychometric parallelism is small, and the p value of χ^2 is large. When there are statistical differences in the

intercorrelations among predictors, the degree of psychometric parallelism is less. In other words, when L approaches 0, the χ^2 value is large, and the p.value of χ^2 is small (Chang, 1996).

The main purpose of this study was to investigate the effect of psychometric parallelism on the error of accuracy for equal weights and least-squares weights. The central research question of this study was: Does psychometric parallelism have the effect on the difference between error of accuracy for least-squares weights and equal weights when there are variations in number of predictors, sample size, and magnitude of intercorrelations?

As stated above, this study was concerned with the error of accuracy of two regression techniques. The error of accuracy can be operationalized in several ways. In this study, it was measured by the difference between the sample R^2 estimated by the target regression techniques and the population R^2. The differences for both least-squares weights and equal weights, denoted d_1 and d_2, are calculated as follows:

$$d_1 = |\rho^2 - R_{ls}^2| \tag{3}$$

$$d_2 = |\rho^2 - R_{ew}^2| \tag{4}$$

where ρ^2 is the population R^2, R_{ls}^2 is the sample R^2 estimated by least-squares weights, and R_{ew}^2 is the sample R^2 estimated by equal weights.

In reality, R_{ls}^2 is usually overestimated and R_{ew}^2 is usually underestimated relative to the level of ρ^2. Therefore, d_1 and d_2 indicate the real deviation between R^2 and ρ^2 for each weighting method respectively. When d_1 is zero, least-squares weights estimate ρ^2

perfectly. The larger the value of d_1, the less accurately least-squares weights perform. The same consideration can be applied to d_2 for equal weights.

The difference between d_1 and d_2, denoted d_3, is calculated as follows:

$$d_3 = d_2 - d_1 \tag{5}$$

Unlike d_1 and d_2, d_3 can have either a positive value, a negative value, or 0. A positive d_3 means that least-squares weights perform more accurately than equal weights because the deviation between R^2_{cw} and ρ^2 is larger than that between R^2_{ls} and ρ^2. On the other hand, when d_3 is negative, equal weights perform more accurately than least-squares weights because the deviation between R^2_{cw} and ρ^2 is smaller than that between R^2_{ls} and ρ^2. Both weighting methods perform equally if d_3 is 0.

Method

The matrix procedure of Interactive Matrix Language (IML) within the Statistical Analysis System (SAS) was used to generate samples from multivariate normal populations with specified population parameters (SAS/IML, 1988). Correlation matrices were constructed for ninety populations which varied according to the mean of intercorrelations among predictors ($\bar{r}$ = 0.2, 0.4, and 0.6), number of predictors (k = 3, 5, and 7), and ten varying patterns of correlation matrices which will be described later. The predictor-criterion validity was .3 which is typical across a wide range of empirical studies (Fiske, 1978; Mischel, 1968). The three different means of intercorrelations were selected in the study because Shcmitt, Coyle, and Rauschenberger (1977) stressed that "the levels of independent variables appeared to be reasonable in light of empirical research (Ghiselli, 1966, 1983) and the levels of intercorrelations and validities typically reported for test

batteries" (p.753). In addition, the correlations between the criterion variable and each predictor should be the same according to the definition of psychometric parallelism (Paunonen & Gardner, 1991).

In this study, ten varying patterns with different correlation matrices but the same mean of intercorrelation were designed in an attempt to obtain matrices where all intercorrelations were from relatively unequal to perfectly equal. In other words, the predictors are from relatively unparallel to extremely parallel. For the $k = 3$ and $\bar{r} = 0.2$ population, three intercorrelations among the predictors were set at 0.02, 0.2, and 0.38 and labeled Pattern 1. In Pattern 2, the intercorrelation of 0.02 was increased to 0.04, the intercorrelation of 0.2 stayed the same, and the intercorrelation of 0.38 was dropped to 0.36. This procedure was repeated, for the following patterns, up to Pattern 10. In other words, one of these three intercorrelations was increased from 0.02 to 0.2 with increments of 0.02 from Patterns 1 through 10. The second one was 0.2 for all ten patterns. The last one was decreased from 0.38 to 0.2 with decrements of 0.02 for each separate simulation.

For the $k = 3$ and $\bar{r} = 0.4$ population, three intercorrelations among the predictors were set at 0.22, 0.4, and 0.58 in Pattern 1. In Pattern 2, the intercorrelation of 0.22 was increased to 0.24, the intercorrelation of 0.4 was still the same, and the intercorrelation of 0.58 was decreased to 0.56. That is, one intercorrelation was increased from 0.22 to 0.4 with increments of 0.02 from Patterns 1 through 10. One of them was 0.4 for all ten patterns. The last one was dropped from 0.58 to 0.4 with decrements of 0.02 for each separate simulation.

For the $k = 3$ and $\bar{r} = 0.6$ population, three intercorrelations among the predictors were set at 0.42, 0.6, and 0.78 in Pattern 1. In Pattern 2, the intercorrelation of 0.42 was

increased to 0.44, another intercorrelation was still 0.6, and the last one was decreased from 0.78 to 0.76. Namely, one of the intercorrelations was increased from 0.42 to 0.6 with increments of 0.02 from Patterns 1 through 10. One of them was 0.6 for all ten patterns. The third intercorrelation was decreased from 0.78 to 0.6 with decrements of 0.02 for each separate simulation. The ten different patterns for each mean of intercorrelations with $k = 3$ are listed in Table 1.

The matrices for Patterns 1, 2, and 10 with $k = 3$ and $\bar{r} = 0.2$, 0.4, and 0.6 are listed in Tables 2, 3, and 4. All matrices in this study are $(k + 1)$ rows by $(k + 1)$ columns. The first row and the first column of the matrices represent the criterion variable. Throughout this study, the matrices are denoted by upper case letter *M*. The first two places in the subscript of the matrices refer to the number of predictors, the next two places refer to the mean of the interrelation, and the last two or three places indicate the pattern of the correlation matrices.

The same consideration was applied to the $k = 5$ population and the $k = 7$ population. The number of intercorrelations among predictors is 10 for a 5-predictor population. For the ten patterns with $\bar{r} = 0.2$, three of the intercorrelations were varied from 0.02 to 0.2 with increments of 0.02 for each pattern, four of them were 0.2 for all patterns, and the last three were varied from 0.38 to 0.2 with decrements of 0.02 for each pattern. All intercorrelations were 0.2 in Pattern 10. For the $\bar{r} = 0.4$ patterns, three of the intercorrelations were varied from 0.22 to 0.4 with increments of 0.02 for each pattern, four of them were 0.4 for all patterns, and the last three were varied from 0.58 to 0.4 with decrements of 0.02 for each pattern. Like the pattern with $\bar{r} = 0.2$, all intercorrelations in Pattern 10 were the same, 0.4. For the $\bar{r} = 0.6$ patterns, three of the intercorrelations were

varied from 0.42 to 0.6 with increments of 0.02 for each pattern, four of them were still 0.6 for all patterns, and the last three were varied from 0.78 to 0.6 with decrements of 0.02 for each pattern. The matrices for Patterns 1, 2, and 10 with $k = 5$ and $\bar{r} = 0.2$, 0.4, and 0.6 are listed in Tables 5, 6, and 7.

For a 7-predictor population, the number of intercorrelations among predictors is 21. For the patterns with $\bar{r} = 0.2$, seven of the intercorrelations were varied from 0.02 to 0.2 with increments of 0.02 from Patterns 1 through 10, seven of them were 0.2 for all patterns, and the last seven were varied from 0.38 to 0.2 with decrements of 0.02 for each pattern. For the patterns with $\bar{r} = 0.4$, seven of the intercorrelations were varied from 0.22 to 0.4 with increments of 0.02 for each pattern, seven of them were 0.4 for all patterns, and the last seven were varied from 0.58 to 0.4 with decrements of 0.02 for each pattern. For the $\bar{r} = 0.6$ patterns, seven of the intercorrelations were varied from 0.42 to 0.6 with increments of 0.02 from Patterns 1 to 10, seven of them were 0.6 for all patterns, and the last seven were varied from 0.78 to 0.6 with decrements of 0.02 for each pattern. All the 21 intercorrelations were the same, 0.6, in the last pattern. The matrices for Patterns 1, 2, and 10 with $k = 7$ and $\bar{r} = 0.2$, 0.4, and 0.6 are provided in Tables 8 through 10.

A total of 90 populations with different characteristics was included: 3 (number of predictors) by 3 (mean of intercorrelations) by 10 (matrix patterns). For each population, six levels of n/k (5/1, 10/1, 20/1, 30/1, 40/1, and 50/1) were considered. Therefore, a total of 540 data sets was involved: 90 (populations) by 6 (n/k levels). Five hundred independent random samples were extracted from each of the 540 data sets. The means of statistics for every extracted five hundred independent random samples in each data set were estimated.

The two weighting methods which were investigated in this study were applied to each sample. The weights from each method were used to obtain the sample R^2 for each method. To find the answer to the central question, the estimated value of d_3 was counted as 1 for equal weights when it was negative or 0. It was counted as 1 for least-squares weights when d_3 was positive. The counts for both weighting methods were cumulated for each of the 540 analyses. The χ^2 goodness of fit test was performed with the observed cumulated frequencies for each analysis.

Results

Tables 11, 12, and 13 provide $\hat{p}$ of χ^2 for testing psychometric parallelism among predictors for each data set. Examination of these tables seems to indicate that $\hat{p}$ decreases as the sample size increases for most of the analyses. However, the $\hat{p}$ values in Patterns 8 through 10 show an unstable phenomenon. In these three patterns, $\hat{p}$ does not always increase as n/k increases. For each ratio of n/k, $\hat{p}$ shows an ascending trend from Patterns 1 through 10. The increase is more obvious when the ratio of n/k is higher. Furthermore, $\hat{p}$ increases as $\bar{r}$ decreases.

Tables 14 through 16 contain the cumulated frequencies for equal weights and least squares weights with different combinations of k, n/k, and $\bar{r}$. The frequency for equal weights is always higher than that for least squares weights when $k = 3$ regardless of the magnitude of $\bar{r}$. Least squares weights show higher frequency in some cases with $k = 5$ or 7. When $k = 5$, the number for least squares weights having higher frequency is 6 for $\bar{r} = 0.2$, 3 for $\bar{r} = 0.4$, and 0 for $\bar{r} = 0.6$ out of the 180 comparisons. When $k = 7$, the number

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for least squares weights having higher frequency is 14, 6, and 3 for $\bar{r}$ = 0.2, 0.4, and 0.6, respectively.

The frequency for equal weights' superiority decreases as *n/k* increases. The decrease tends to be more obvious when *k* = 7 than when *k* = 3. For example, in Pattern 1 with *k* = 3 and $\bar{r}$ = 0.2, the frequency for equal weights is 328, 302, 290, 269, 271, and 265 for *n/k* = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1. In the same pattern and same $\bar{r}$ but with *k* = 7, the frequency for equal weights is 326, 275, 217, 193, 205, and 184 for *n/k* = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1, respectively.

The tendency of the frequency for equal weights' superiority is not consistent as the matrix pattern changes from 1 to 10 when *k* = 3 or 5. However, the frequency for equal weights' superiority does smoothly increase along with the increase of the matrix pattern when *k* = 7. In addition, the frequency for equal weights tends to decrease more quickly when the pattern's number is lower. For example, in Pattern 1 with *k* = 7 and $\bar{r}$ = 0.2, the frequency for equal weights is 326, 275, 217, 193, 205, and 184 for *n/k* = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1, respectively. In Pattern 9 with the same *k* and $\bar{r}$, the frequency for equal weights is 357, 318, 282, 289, 274, and 269 for *n/k* = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1, respectively.

Inspection of the distribution of these frequencies in Tables 15 and 16 reveals that those least squares weights having higher frequency than equal weights are only found in Patterns 1 through 5. The number of least squares weights having higher frequency than equal weights increases as *n/k* increases. When *k* = 7 and $\bar{r}$ = 0.2, the number of least squares weights having higher frequency than equal weights is 0, 0, 2, 3, 4, and 5 for *n/k* = 5/1, 10/1, 20/1, 30/1, 40/1, and 5/1, respectively. The bold frequencies with the assorted

combinations of k and $\bar{r}$ in Tables 15 and 16 seem to resemble different sizes of right-angled triangles. The lower the magnitude of $\bar{r}$ is, the longer the side of the triangle. In addition, the higher the number of k's is, the longer the side of the triangle.

To guide the evaluation of the accuracy measures by equal weights and least squares weights, the χ^2 goodness of fit test was performed on the observed frequencies for each data set. The null hypothesis is that 50 percent of equal weights would perform more accurately than least squares weights in estimating ρ^2, and 50 percent of equal weights would not. Since the total frequency for each analysis is 500, the expected frequency is 250 for each weighting method.

Across the different patterns, k, and $\bar{r}$, all the null hypotheses are rejected when n/k = 5/1 or 10/1 because the observed frequencies for equal weights are much higher than the expected. The number of rejected null hypotheses becomes less as the ratio of n/k increases. From n/k = 20 to 50 most of the rejections, due to unusually high frequencies for equal weights, occur in the relatively high-number patterns which have more homogeneous intercorrelations.

The rejection of the null hypothesis, due to unlikely high frequencies for least squares weights, only happens in the first several patterns with $k = 7$, $\bar{r} = 0.2$, and n/k larger than 10/1. Besides, there are some cases with $k = 5$, $\bar{r} = 0.4$ or 0.6, and the pattern's number less than 5 in which the frequencies for least squares weights are higher than those for equal weights. None of them, however, is statistically significant.

Overall, Tables 11 through 16 are useful for clarifying that the psychometric parallelism has a tendency to affect the error of accuracy for both equal weights and least-squares weights in different ways. First, the higher the number of predictors, the

higher the minimum p value of χ^2 for testing the psychometric parallelism is needed to prove equal weights less error of accuracy than least-squares weights. That is, the equality among intercorrelations is more essential and critical for using equal weights when the number of predictors is higher.

Second, the higher the observation-to-predictor ratio, the higher the minimum p value is needed to prove equal weights better than least squares. Equal weights are always better when the ratio of observation to predictor is less than or equal to 10 for any level of psychometric parallelism. This finding does correspond to that of Einhorn and Hogarth (1975), in which equal weights are superior to least squares weights when sample size is small.

Finally, the higher the magnitude of the mean of intercorrelation is, the lower the minimum p value is needed to prove equal weights less error of accuracy than least squares weights. There is no argument that equal weights tend to be more powerful when the value of $\bar{r}$ is higher. However, even with a low mean of intercorrelation, equal weights can still be better than least squares weights if the ratio of observation to predictor is small or the level of psychometric parallelism is relatively high.

Conclusions

The results indicate that equal weights always perform more precisely than least-squares weights as long as the following situations are satisfied: (a) the number of predictors is small, 3 , (b) the ratio of observation to predictor is small, less than or equal to 10, and (c) the magnitude of the mean of intercorrelation is high, at least 0.6. Least-squares weights, on the other hand, may perform more accurately than equal weights with the

opposite combination: (a) a large number of predictors, (b) high ratio of observation to predictor, and (c) low intercorrelations.

Nevertheless, the combination of a large number of predictors, large sample sizes, and a low mean of intercorrelation does not guarantee that least-squares weights are more accurate than equal weights. Based on the results for the combination of $k = 7$, $\bar{r} = 0.2$, and $n/k = 50$, equal weights are still more accurate than least-squares weights for Patterns 6 through 10. In this research design, as the pattern's number increases, the likelihood ratio of psychometric parallelism increases. This is evidence that the variability of intercorrelations, level of psychometric parallelism, is an important element to be considered. It is involved in addition to the number of predictors, sample size, and magnitude of intercorrelation in determining the error of accuracy between equal weights and least-squares weights.

It should be emphasized that the present paper is not relevant to the other main use of the least-squares weights procedure in hypothesis testing (e.g. testing whether a model is statistically significant, whether a particular regression coefficient is significantly positive, etc.). In other words, if in a particular sample data with high level of psychometric parallelism so that the equal weights would be expected to be better in predictive power than least-squares weights, this fact does not nullify the value of the multiple regression results in testing hypotheses.

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Table 1

Ten Patterns of the Intercorrelations for k = 3 and $\bar{r}$ = 0.2, 0.4, and 0.6

Pattern	$\bar{r}$ = 0.2	$\bar{r}$ = 0.4	$\bar{r}$ = 0.6	Interval	Range
1	.02 .20 .38	.22 .40 .58	.42 .60 .78	.18	.36
2	.04 .20 .36	.24 .40 .56	.44 .60 .76	.16	.32
3	.06 .20 .34	.26 .40 .54	.46 .60 .74	.14	.28
4	.08 .20 .32	.28 .40 .52	.48 .60 .72	.12	.24
5	.10 .20 .30	.30 .40 .50	.50 .60 .70	.10	.20
6	.12 .20 .28	.32 .40 .48	.52 .60 .68	.08	.16
7	.14 .20 .26	.34 .40 .46	.54 .60 .66	.06	.12
8	.16 .20 .24	.36 .40 .44	.56 .60 .64	.04	.08
9	.18 .20 .22	.38 .40 .42	.58 .60 .62	.02	.04
10	.20 .20 .20	.40 .40 .40	.60 .60 .60	.00	.00

Table 2

The Matrices for Patterns 1, 2, and 10 with k = 3 and $\bar{r}$ = 0.2

M_{k3r2p1}	M_{k3r2p2}	$M_{k3r2p10}$
1 .30 .30 .30	1 .30 .30 .30	1 .30 .30 .30
.30 1 .02 .20	.30 1 .04 .20	.30 1 .20 .20
.30 .02 1 .38	.30 .04 1 .36	.30 .20 1 .20
.30 .20 .38 1	.30 .20 .36 1	.30 .20 .20 1

Table 3

The Matrices for Patterns 1, 2, and 10 with k = 3 and $\bar{r}$ = 0.4

M_{k3r4p1}	M_{k3r4p2}	$M_{k3r4p10}$
1 .30 .30 .30	1 .30 .30 .30	1 .30 .30 .30
.30 1 .22 .40	.30 1 .24 .40	.30 1 .40 .40
.30 .22 1 .58	.30 .24 1 .56	.30 .40 1 .40
.30 .40 .58 1	.30 .40 .56 1	.30 .40 .40 1

Table 4

The Matrices for Patterns 1, 2, and 10 with k = 3 and $\bar{r}$ = 0.6

M_{k3r6p1}	M_{k3r6p2}	$M_{k3r6p10}$
1 .30 .30 .30	1 .30 .30 .30	1 .30 .30 .30
.30 1 .42 .60	.30 1 .44 .60	.30 1 .60 .60
.30 .42 1 .78	.30 .44 1 .76	.30 .60 1 .60
.30 .60 .78 1	.30 .60 .76 1	.30 .60 .60 1

Table 5

The Matrices for Patterns 1, 2, and 10 with k = 5 and $\bar{r}$ = 0.2

M_{k5r2p1}						M_{k5r2p2}						$M_{k5r2p10}$					
1	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30
.30	1	.02	.02	.02	.20	.30	1	.04	.04	.04	.20	.30	1	.20	.20	.20	.20
.30	.02	1	.20	.20	.20	.30	.04	1	.20	.20	.20	.30	.20	1	.20	.20	.20
.30	.02	.20	1	.38	.38	.30	.04	.20	1	.36	.36	.30	.20	.20	1	.20	.20
.30	.02	.20	.38	1	.38	.30	.04	.20	.36	1	.36	.30	.20	.20	.20	1	.20
.30	.20	.20	.38	.38	1	.30	.20	.20	.36	.36	1	.30	.20	.20	.20	.20	1

Table 6

The Matrices for Patterns 1, 2, and 10 with k = 5 and $\bar{r}$ = 0.4

M_{k5r4p1}						M_{k5r4p2}						$M_{k5r4p10}$					
1	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30
.30	1	.22	.22	.22	.40	.30	1	.24	.24	.24	.40	.30	1	.40	.40	.40	.40
.30	.22	1	.40	.40	.40	.30	.24	1	.40	.40	.40	.30	.40	1	.40	.40	.40
.30	.22	.40	1	.58	.58	.30	.24	.40	1	.56	.56	.30	.40	.40	1	.40	.40
.30	.22	.40	.58	1	.58	.30	.24	.40	.56	1	.56	.30	.40	.40	.40	1	.40
.30	.40	.40	.58	.58	1	.30	.40	.40	.56	.56	1	.30	.40	.40	.40	.40	1

Table 7

The Matrices for Patterns 1, 2, and 10 with k = 5 and $\bar{r}$ = 0.6

M_{k5r6p1}	M_{k5r6p2}	$M_{k5r6p10}$
1 .30 .30 .30 .30 .30	1 .30 .30 .30 .30 .30	1 .30 .30 .30 .30 .30
.30 1 .42 .42 .42 .60	.30 1 .44 .44 .44 .60	.30 1 .60 .60 .60 .60
.30 .42 1 .60 .60 .60	.30 .44 1 .60 .60 .60	.30 .60 1 .60 .60 .60
.30 .42 .60 1 .78 .78	.30 .44 .60 1 .76 .76	.30 .60 .60 1 .60 .60
.30 .42 .60 .78 1 .78	.30 .44 .60 .76 1 .76	.30 .60 .60 .60 1 .60
.30 .60 .60 .78 .78 1	.30 .60 .60 .76 .76 1	.30 .60 .60 .60 .60 1

Table 8

The Matrices for Patterns 1, 2, and 10 with k = 7 and $\bar{r}$ = 0.2

M_{k7r2p1}								M_{k7r2p2}								$M_{k7r2p10}$							
1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30
.30	1	.02	.02	.02	.02	.02	.02	.30	1	.04	.04	.04	.04	.04	.04	.30	1	.20	.20	.20	.20	.20	.20
.30	.02	1	.02	.20	.20	.20	.20	.30	.04	1	.04	.20	.20	.20	.20	.30	.20	1	.20	.20	.20	.20	.20
.30	.02	.02	1	.20	.20	.20	.38	.30	.04	.04	1	.20	.20	.20	.36	.30	.20	.20	1	.20	.20	.20	.20
.30	.02	.20	.20	1	.38	.38	.38	.30	.04	.20	.20	1	.36	.36	.36	.30	.20	.20	.20	1	.20	.20	.20
.30	.02	.20	.20	.38	1	.38	.38	.30	.04	.20	.20	.36	1	.36	.36	.30	.20	.20	.20	.20	1	.20	.20
.30	.02	.20	.20	.38	.38	1	.38	.30	.04	.20	.20	.36	.36	1	.36	.30	.20	.20	.20	.20	.20	1	.20
.30	.02	.20	.38	.38	.38	.38	1	.30	.04	.20	.36	.36	.36	.36	1	.30	.20	.20	.20	.20	.20	.20	1

Table 9

The Matrices for Patterns 1, 2, and 10 with k = 7 and $\bar{r}$ = 0.4

M_{k7r4p1}								M_{k7r4p2}								$M_{k7r4p10}$							
1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30
.30	1	.22	.22	.22	.22	.22	.22	.30	1	.24	.24	.24	.24	.24	.24	.30	1	.40	.40	.40	.40	.40	.40
.30	.22	1	.22	.40	.40	.40	.40	.30	.24	1	.24	.40	.40	.40	.40	.30	.40	1	.40	.40	.40	.40	.40
.30	.22	.22	1	.40	.40	.40	.58	.30	.24	.24	1	.40	.40	.40	.56	.30	.40	.40	1	.40	.40	.40	.40
.30	.22	.40	.40	1	.58	.58	.58	.30	.24	.40	.40	1	.56	.56	.56	.30	.40	.40	.40	1	.40	.40	.40
.30	.22	.40	.40	.58	1	.58	.58	.30	.24	.40	.40	.56	1	.56	.56	.30	.40	.40	.40	.40	1	.40	.40
.30	.22	.40	.40	.58	.58	1	.58	.30	.24	.40	.40	.56	.56	1	.56	.30	.40	.40	.40	.40	.40	1	.40
.30	.22	.40	.58	.58	.58	.58	1	.30	.24	.40	.56	.56	.56	.56	1	.30	.40	.40	.40	.40	.40	.40	1

Table 10

The Matrices for Patterns 1, 2, and 10 with k = 7 and $\bar{r}$ = 0.6

M_{k7r6p1}								M_{k7r6p2}								$M_{k7r6p10}$							
1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30	1	.30	.30	.30	.30	.30	.30	.30
.30	1	.42	.42	.42	.42	.42	.42	.30	1	.44	.44	.44	.44	.44	.44	.30	.60	.60	.60	.60	.60	.60	.60
.30	.42	1	.42	.60	.60	.60	.60	.30	.44	1	.44	.60	.60	.60	.60	.30	.60	1	.60	.60	.60	.60	.60
.30	.42	.42	1	.60	.60	.60	.78	.30	.44	.44	1	.60	.60	.60	.76	.30	.60	.60	1	.60	.60	.60	.60
.30	.42	.60	.60	1	.78	.78	.78	.30	.44	.60	.60	1	.76	.76	.76	.30	.60	.60	.60	1	.60	.60	.60
.30	.42	.60	.60	.78	1	.78	.78	.30	.44	.60	.60	.76	1	.76	.76	.30	.60	.60	.60	.60	1	.60	.60
.30	.42	.60	.60	.78	.78	1	.78	.30	.44	.60	.60	.76	.76	1	.76	.30	.60	.60	.60	.60	.60	1	.60
.30	.42	.60	.78	.78	.78	.78	1	.30	.44	.60	.76	.76	.76	.76	1	.30	.60	.60	.60	.60	.60	.60	1

Table 11

The $\hat{p}$ of χ^2 for Difference of Psychometric Parallelism for Each Sample with k = 3; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k	*n*					$\bar{r}$ = .2					
5/1	15	.7148	.7581	.7351	.7649	.7954	.7955	.8188	.7957	.8098	.8497
10/1	30	.6404	.6742	.7147	.7607	.7769	.8239	.8160	.8352	.8465	.8533
20/1	60	.4770	.5193	.5915	.6868	.7367	.8038	.8049	.8367	.8608	.8611
30/1	90	.2927	.3811	.5052	.5768	.6669	.7460	.8117	.8395	.8704	.8818
40/1	120	.1890	.2120	.3823	.5294	.6124	.6904	.7825	.8408	.8436	.8623
50/1	150	.1247	.2260	.3087	.4244	.5509	.6854	.7424	.8435	.8629	.8730
						$\bar{r}$ = .4					
5/1	15	.6269	.6717	.7244	.7343	.7818	.7886	.8149	.8253	.8165	.7937
10/1	30	.4873	.5745	.6212	.6924	.7380	.7723	.8199	.8299	.8473	.8583
20/1	60	.2160	.3388	.4449	.5568	.6427	.7293	.7529	.8245	.8499	.8855
30/1	90	.1235	.1957	.2882	.4117	.5294	.6651	.7465	.8124	.8501	.8753
40/1	120	.0430	.1192	.2072	.3188	.4607	.5846	.6886	.8091	.8333	.8675
50/1	150	.0187	.0500	.1308	.2263	.3794	.5250	.6622	.7833	.8533	.8662
						$\bar{r}$ = .6					
5/1	15	.3983	.5012	.5801	.6310	.6961	.7456	.7983	.8175	.8295	.8156
10/1	30	.1646	.2709	.3811	.5236	.5996	.7062	.7590	.8219	.8342	.8516
20/1	60	.0255	.0629	.1511	.2576	.4033	.5346	.6508	.8068	.8387	.8609
30/1	90	.0033	.0111	.0496	.1230	.2838	.4164	.5886	.7453	.8383	.8571
40/1	120	.0002	.0022	.0169	.0647	.1678	.3197	.5444	.7180	.8532	.8713
50/1	150	.0001	.0009	.0043	.0325	.0963	.2660	.4754	.6763	.8317	.8629

Table 12

The $\hat{p}$ of χ^2 for Difference of Psychometric Parallelism for Each Sample with k = 5; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k	*n*					$\bar{r}$ = .2					
5/1	25	.5698	.5941	.7457	.6978	.7291	.7494	.7686	.7870	.7948	.8075
10/1	50	.3901	.4765	.5334	.6291	.6908	.7319	.7884	.8236	.8398	.8356
20/1	100	.1156	.2272	.2848	.4296	.5539	.6636	.7487	.8074	.8382	.8542
30/1	150	.0357	.0804	.1511	.2735	.4326	.5696	.6932	.7610	.8516	.8591
40/1	200	.0051	.0288	.0754	.1572	.2966	.4688	.6443	.7646	.8361	.8751
50/1	250	.0010	.0063	.0264	.0912	.2012	.3758	.5392	.7413	.8196	.8714
						$\bar{r}$ = .4					
5/1	25	.4611	.5270	.5640	.6389	.6782	.7244	.7710	.7909	.7921	.7691
10/1	50	.2105	.2981	.4119	.5085	.6312	.6926	.7471	.8061	.8297	.8381
20/1	100	.0319	.0787	.1545	.2839	.4249	.5677	.7075	.7987	.8293	.8474
30/1	150	.0021	.0116	.0516	.1111	.2723	.4356	.6189	.7379	.8369	.8402
40/1	200	.0001	.0021	.0125	.0477	.1633	.3269	.5291	.6974	.8340	.8639
50/1	250	.0000	.0003	.0031	.0281	.0868	.2290	.4420	.6774	.8138	.8675
						$\bar{r}$ = .6					
5/1	25	.1631	.2779	.3514	.5015	.5571	.6553	.7421	.7667	.7750	.7956
10/1	50	.0217	.0600	.1438	.2832	.4214	.5486	.6807	.7176	.8068	.8514
20/1	100	.0000	.0006	.0093	.0485	.1544	.3119	.5156	.7079	.8166	.8622
30/1	150	.0000	.0000	.0003	.0080	.0417	.1746	.3839	.6180	.8122	.8540
40/1	200	.0000	.0000	.0000	.0004	.0143	.0882	.2686	.5806	.7875	.8519
50/1	250	.0000	.0000	.0000	.0000	.0030	.0285	.1808	.5235	.7845	.8551

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Table 13

The $\hat{p}$ of χ^2 for Difference of Psychometric Parallelism for Each Sample with k = 7; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k	*n*					$\bar{r}$ = .2					
5/1	35	.3700	.4275	.5004	.5915	.6394	.6723	.7394	.7624	.8116	.7984
10/1	70	.1127	.2034	.2902	.4119	.5291	.6438	.7106	.7931	.8297	.8501
20/1	140	.0042	.0283	.0614	.1426	.2979	.4538	.6202	.7298	.8286	.8395
30/1	210	.0001	.0019	.0294	.0363	.1241	.2653	.5099	.6858	.7932	.8444
40/1	280	.0000	.0001	.0009	.0067	.0487	.1612	.4082	.6046	.7856	.8519
50/1	350	.0000	.0000	.0001	.0011	.0016	.0903	.2697	.5592	.7583	.8409
						$\bar{r}$ = .4					
5/1	35	.2118	.2907	.4376	.4861	.5932	.6459	.6997	.7560	.7755	.8033
10/1	70	.0416	.0949	.1686	.2921	.4082	.5342	.6800	.7571	.7982	.8548
20/1	140	.0001	.0037	.0140	.0574	.1418	.3230	.5330	.7043	.7903	.8379
30/1	210	.0000	.0000	.0005	.0093	.0404	.1672	.3475	.6011	.7905	.8549
40/1	280	.0000	.0000	.0000	.0004	.0106	.0630	.2364	.5243	.7647	.8507
50/1	350	.0000	.0000	.0000	.0000	.0019	.0244	.1858	.4564	.7187	.8401
						$\bar{r}$ = .6					
5/1	35	.0201	.0733	.1547	.2588	.3959	.5276	.6429	.7281	.8035	.7810
10/1	70	.0001	.0034	.0132	.0647	.1445	.3412	.5294	.6962	.7872	.8282
20/1	140	.0000	.0000	.0000	.0007	.0141	.0831	.2826	.5472	.7623	.8583
30/1	210	.0000	.0000	.0000	.0000	.0003	.0210	.1329	.4177	.7480	.8680
40/1	280	.0000	.0000	.0000	.0000	.0000	.0017	.0466	.3136	.6898	.8572
50/1	350	.0000	.0000	.0000	.0000	.0000	.0002	.0187	.2376	.6418	.8284

Table 14

The Counts of $\hat{d}_3$ for Equal Weights and Least Squares Weights with k = 3; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k						$\bar{r}$ = .2					
5/1	EW	328**	330**	311**	355**	325**	333**	339**	335**	336**	342**
	LS	172	170	189	145	175	167	161	165	164	158
10/1	EW	302	313**	302**	288**	329**	309**	310**	301**	290**	303**
	LS	198	187	198	212	171	191	190	199	210	197
20/1	EW	290**	286**	289**	285**	301**	278*	295**	270	287**	289**
	LS	210	214	211	215	199	222	205	230	213	211
30/1	EW	269	287**	287**	260	292**	267	282**	278*	279**	269
	LS	231	213	213	240	208	233	218	222	221	231
40/1	EW	271	268	267	264	278*	279**	254	267	263	277*
	LS	229	232	233	236	222	221	246	233	237	223
50/1	EW	265	266	268	272*	273*	256	275*	271	276*	272*
	LS	235	234	232	228	227	244	225	229	224	228
						$\bar{r}$ = .4					
5/1	EW	369**	352**	346**	363**	344**	345**	339**	348**	361**	346**
	LS	131	148	154	137	156	155	161	152	139	154
10/1	EW	292**	298**	322**	299**	314**	277*	295**	298**	305**	312**
	LS	208	202	178	201	186	222	205	202	195	188
20/1	EW	282**	286**	293**	299**	279**	275*	308**	295**	284**	305**
	LS	218	214	207	201	221	225	192	205	216	195
30/1	EW	275*	276*	290**	290**	288**	262	280**	281**	287**	275*
	LS	225	224	210	210	212	238	220	219	213	225
40/1	EW	261	284**	287**	280**	258	259	301**	272*	294**	265
	LS	239	216	213	220	242	241	199	228	206	235
50/1	EW	271	265	260	293**	267	273*	290**	289**	285**	274*
	LS	229	235	240	207	233	227	210	211	215	226
						$\bar{r}$ = .6					
5/1	EW	353**	351**	351**	369**	360**	366**	345**	363**	367**	361**
	LS	147	149	149	131	140	134	155	137	133	139
10/1	EW	313**	299**	314**	315**	297**	309**	335**	313**	319**	302**
	LS	187	201	186	185	203	191	165	177	181	198
20/1	EW	264	283**	280**	286**	282**	281**	292**	301**	286**	293**
	LS	236	217	220	214	218	219	208	199	214	207
30/1	EW	272*	287**	274*	286**	290**	265	277*	282**	269	274*
	LS	228	213	226	214	210	235	223	218	231	226
40/1	EW	286**	278*	258	280**	286**	265	271	266	268	262
	LS	214	222	242	220	214	235	229	234	232	238
50/1	EW	260	277*	287**	286**	290**	270	271	271	283**	260
	LS	240	223	213	214	210	230	229	229	217	240

Note. EW = equal weights, LS = least squares weights.
* $p < 0.05$. ** $p < 0.01$.

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Table 15

The Counts of $\hat{d}_3$ for Equal Weights and Least Squares Weights with k = 5; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k						$\bar{r}$ = .2					
5/1	EW	345**	335**	348**	356**	330**	350**	332**	340**	348**	340**
	LS	155	165	152	144	170	150	168	160	152	160
10/1	EW	313**	301**	318**	308**	302**	322**	321**	317**	322**	318**
	LS	187	199	182	192	198	178	179	183	178	182
20/1	EW	259	281**	291**	286**	284**	289**	308**	316**	294**	299**
	LS	241	219	209	214	216	211	192	194	206	201
30/1	EW	**240**	254	285**	282**	278*	293**	275*	284**	273*	276*
	LS	**260**	246	215	218	222	207	225	216	227	224
40/1	EW	**244**	**232**	276*	273*	273*	278*	287**	274*	275*	277*
	LS	**256**	**268**	224	227	227	222	213	226	225	223
50/1	EW	**236**	**243**	**242**	265	259	270	274*	292**	267	283**
	LS	**264**	**257**	**258**	235	241	230	226	208	233	217
						$\bar{r}$ = .4					
5/1	EW	371**	363**	350**	376**	350**	358**	355**	356**	349**	376**
	LS	129	137	150	124	150	142	145	144	151	124
10/1	EW	320**	322**	326**	327**	323**	324**	325**	339**	322**	319**
	LS	180	178	174	173	177	176	175	161	178	181
20/1	EW	277*	311**	296**	291**	278*	311**	308**	319**	315**	307**
	LS	223	189	204	209	222	189	192	181	185	193
30/1	EW	255	288**	278*	289**	299**	310**	297**	289**	293**	282**
	LS	245	212	222	211	201	190	203	211	207	218
40/1	EW	**247**	270	254	266	285**	277*	262	278*	299**	273*
	LS	**253**	230	246	234	215	223	238	222	201	227
50/1	EW	**240**	**245**	262	273*	272*	265	269	275*	294**	285**
	LS	**260**	**255**	238	227	228	235	231	225	206	215
						$\bar{r}$ = .6					
5/1	EW	393**	388**	395**	390**	391**	400**	377**	370**	390**	380**
	LS	107	112	105	110	109	100	123	130	110	120
10/1	EW	333**	324**	345**	332**	338**	328**	352**	335**	354**	332**
	LS	167	176	155	168	162	172	148	165	146	168
20/1	EW	291**	302**	304**	308**	323**	303**	319**	314**	308**	314**
	LS	209	198	196	192	177	197	181	186	192	186
30/1	EW	268	268	267	301**	283**	294**	300**	293**	297**	285**
	LS	232	232	233	199	217	206	200	207	203	215
40/1	EW	259	267	266	285**	284**	289**	283**	294**	283**	281**
	LS	241	233	234	215	216	211	217	206	217	219
50/1	EW	251	259	265	278*	283**	268	299**	292**	283**	287**
	LS	249	241	235	222	217	232	201	208	217	213

Note. EW = equal weights, LS = least squares weights.
* $p < 0.05$. ** $p < 0.01$.

Table 16

The Counts of $\hat{d}_3$ for Equal Weights and Least Squares Weights with k = 7; $\bar{r}$ = 0.2, 0.4, and 0.6; and n/k = 5/1, 10/1, 20/1, 30/1, 40/1, and 50/1

Pattern		1	2	3	4	5	6	7	8	9	10
n/k						$\bar{r}$ = .2					
5/1	EW	326**	332**	337**	362**	349**	336**	355**	345**	357**	352**
	LS	174	168	163	138	151	163	145	155	143	148
10/1	EW	275*	298**	289**	310**	302**	308**	312**	321**	318**	342**
	LS	225	202	211	190	198	192	188	179	182	158
20/1	EW	**217****	**248**	254	269	287**	306**	276*	304**	282**	319**
	LS	**283**	**252**	246	231	213	194	224	196	218	181
30/1	EW	**193****	**232**	**247**	258	275*	270	276*	283**	289**	278*
	LS	307	**268**	**253**	242	225	230	224	217	212	222
40/1	EW	**205****	**242**	**226***	**244**	261	260	274*	279**	274*	282**
	LS	**295**	**258**	**274**	**256**	239	240	226	221	226	218
50/1	EW	**184****	**211****	**225****	**231**	**248**	262	272*	270	269	285**
	LS	**316**	**289**	**275**	**269**	**253**	238	228	230	231	215
						$\bar{r}$ = .4					
5/1	EW	358**	378**	372**	370**	377**	379**	386**	369**	389**	386**
	LS	142	122	128	130	123	121	114	131	111	114
10/1	EW	304**	319**	328**	345**	333**	343**	340**	340**	346**	347**
	LS	196	181	172	155	167	157	160	160	154	153
20/1	EW	270	277*	281**	288**	310**	295**	303**	320**	311**	307**
	LS	230	223	219	212	190	205	197	180	189	193
30/1	EW	265	267	264	281	297**	288**	280**	317**	309**	291**
	LS	235	233	236	219	203	212	220	183	191	209
40/1	EW	**240**	**247**	270	265	263	281**	298**	278*	276*	291**
	LS	**260**	**253**	230	235	237	219	202	222	224	209
50/1	EW	**231**	**235**	**240**	**249**	278*	272*	275*	280**	268	288**
	LS	**269**	**265**	**260**	**251**	222	228	225	220	232	212
						$\bar{r}$ = .6					
5/1	EW	391**	403**	396**	395**	410**	406**	401**	396**	388**	415**
	LS	109	97	104	105	90	94	99	104	112	85
10/1	EW	333**	341**	336**	359**	342**	366**	344**	348**	354**	334**
	LS	167	159	164	141	158	134	156	152	146	166
20/1	EW	295**	299**	285**	315**	321**	312**	302**	336**	310**	310**
	LS	205	201	215	185	179	188	198	164	190	190
30/1	EW	280**	286**	280**	299**	295**	291**	303**	312**	298**	307**
	LS	220	214	220	201	205	209	197	188	202	193
40/1	EW	**240**	277*	272*	293**	293**	289**	290**	297**	300**	274*
	LS	**260**	223	228	207	207	211	210	203	200	226
50/1	EW	**247**	**249**	268	270	256	296**	290**	309**	279**	305**
	LS	**253**	**251**	232	230	244	204	210	191	221	195

Note. EW = equal weights, LS = least squares weights.
* $p < 0.05$. ** $p < 0.01$.